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ABSTRACT

The stack up of losses within a compressor system gives an indication of its efficiency. In a refrigerator compressor, valves contribute to thermodynamic losses (pressure and cooling capacity) due to valve dynamics and mistiming. This paper proposes an improvement to an existing analytic closed form solution model for efficiency estimation of such compressors by incorporating more detailed valve physics/ dynamics.

For maximum energy efficiency ratio (EER), it is beneficial for variable capacity compressor architectures to drive the piston at a resonant frequency. In an oscillating system, this is typically determined by the mechanical spring-damper characteristics. This resonant frequency is usually a complex function of the geometry and operating conditions due to the gas-spring effect. A closed form solution for performance estimation of such a configuration proposed in literature [Choe and Kim (2000)] does not account for the effect of valve dynamics. However, the timing of valve operation influences the in-cylinder pressure build-up transients and thus modulates the gas-spring stiffness. While an accurate estimation of the resonant frequency requires a multi-physics simulation of the compressor, a detailed simulation of such complexity is time intensive especially when performing design optimization studies and hence analytical models would be preferred. The current work presents an improved equivalent analytic model for such optimization. Uncertainty analysis of the present approach is also discussed by comparing different performance parameters against full non-linear model estimates.

Keywords: compressor, compressor loss, valve, valve dynamics, gas-spring, resonant frequency, energy efficiency ratio

1. INTRODUCTION

The architecture of an electro-dynamically oscillating compressor for refrigeration is essentially a spring-mass damper system which is actuated by the electromagnetic motor. The moving system here is the total moving mass of the reciprocating components which includes the piston, the moving part of the motor and the moving mass of springs attached to the reciprocating components. The spring force is provided by the mechanical springs, and the motion of the moving mass is damped due to friction and gas damping. The gas within the compression chamber acts to resist the piston motion during compression stroke and assists during the expansion stroke. As the piston moves, the force driving it is a net of the spring, damping, motor and the gas force. Considering the entire range of traversal, the gas force is highly non-linear due to different stages of mass and energy exchange within the compressor chamber. An accurate modeling of the physics hence requires a transient simulation, and an analytic closed form solution for compressor performance estimation is difficult without extensive assumptions. One approach to model gas force is to consider it as an equivalent spring (gas spring effect) and an explanation to this assumption is available in Choe and Kim (2000), and Pollak *et al.* (1978).

For the energy efficiency of such a spring-mass system to be a maximum, the system needs to be actuated at the resonant frequency of the system. Operating at an off-resonance point leads to more energy consumption for the desired flow rate. During the discharge stroke, the linear compressor is compressing the refrigerant in the cylinder control volume and the discharge valve opens as the gas pressure exceeds the valve cracking pressure. This is thus an open control volume and the cyclic process is non-linear. As mentioned above, the gas force acting on the moving mass actuated by the motor can be approximated to a spring force (the gas spring effect) which is non-linear through the piston stroke and depends on the operating conditions. Hence there is no fixed resonant frequency. For such a system, this article defines the resonant frequency as the operating frequency that gives maximum EER. The EER for a refrigerator compressor is defined as the ratio of the cooling capacity [BTU/hr] offered and the input power [W] needed to operate it at this load. This paper evaluates the approximation of gas force as a gas spring in the analytical modeling of a linear compressor to estimate its performance. Further, a formulation to model valve loss and updating the mathematical model developed by Choe and Kim (2000) is discussed.

2. FORMULATION

The electro-dynamic oscillating compressor is simulated as a system which is provided a sinusoidal voltage that actuates the motor imparting the piston its reciprocating dynamics. The piston dynamics, valve geometry parameters, heat transfer mechanisms, leakage parameters and refrigerant state properties at suction and discharge dictate the mass flow rate delivered and the energy required by the compressor at this operating voltage. Given the non-linear nature and multiple interacting phenomena, there is no closed form solution for the resonant frequency. Besides, for a target cooling capacity, the voltage amplitude needs to be set appropriately. Hence the combination of the voltage amplitude and operating frequency would give us a maximum EER at the desired cooling capacity. One current practice is to sequentially sweep (incremental steps) for the operating frequency and then sweep the voltage amplitude for desired capacity. Depending on the control approach (for example target head clearance instead of cooling capacity) sweeping for two parameters simultaneously is required. When running a Design of Experiments (DoE) however, in our experience, this control algorithm is not robust and is time intensive. Hence, an analytical formulation [Choe and Kim (2000)] which offers a closed form performance evaluation of a linear compressor is explored. This formulation is hereafter referred to as the baseline model.

The baseline model is developed towards a closed form solution for the linear compressor operating parameters. Based on the operating pressure conditions and geometry parameters, the performance of the compressor can be represented by the baseline model based on assumptions. The three main assumptions are as follows:

- *The cyclic gas dynamics is ideal*; i.e. the suction and discharge process are at constant pressure. This assumption ignores the valve dynamics and reduces the non-linearity of the problem. In reality, valve actuation requires a pressure drop across the valve and during the suction/discharge process pressure fluctuations are present. The valve dynamics depend on the pressure evolution within the cylinder and while capturing these in simulation, reaching a steady state requires time.
- *The voltage and current seen by the linear motor are perfect sinusoids.* In reality, the current evolves based on the ILC circuit of inverter and piston motion. The measurements on an existing compressor showed the current to be not a perfect sinusoid.
- *The piston motion is taken to be a sinusoidal one.* In actual operation, the steady state piston motion may not be sinusoidal as it is based on the input current profile and gas dynamics.

A detailed model which does not assume the above and models realistic conditions is presented elsewhere [Mantri *et. al* (2014)]. Under the assumptions of the baseline model, the performance parameters (resonant frequency, voltage and current amplitude, and the EER) can be related to the motor and cylinder geometry parameters by a closed form equation. The literature formulation detailed in Choe and Kim (2000) predicts the performance parameters for a given piston stroke and head clearance. Thus the resonant frequency is a function Φ such that:

$$F_{resonant} = \Phi(x_{clearance}, Stroke) \qquad 1$$

Considering ideal valve behavior, the mass flow rate delivered by the compressor is dependent on the stroke and head clearance and is given by.

$$\dot{m} = F_{resonant} \cdot \rho_{suction} \cdot A_{piston} \left(Stroke - \left(PR^{1/\gamma} - 1\right) \cdot x_{clearance}\right) \qquad 2$$

where $\rho_{suction}$, A_{piston}, PR and γ are the suction gas density, piston area, pressure ratio and polytropic exponent respectively. For an assumed head clearance and a target cooling flow rate, we have 2 equations with 2 unknowns ($F_{resonant}$, $Stroke$) and simultaneously solving these give a solution set. The head clearance and stroke are also related to L_0 (starting piston mean position) and considering this parameter as part of the DoE set, narrows down the solution. A simple algorithm to solve for the unknowns is shown in Figure 1.

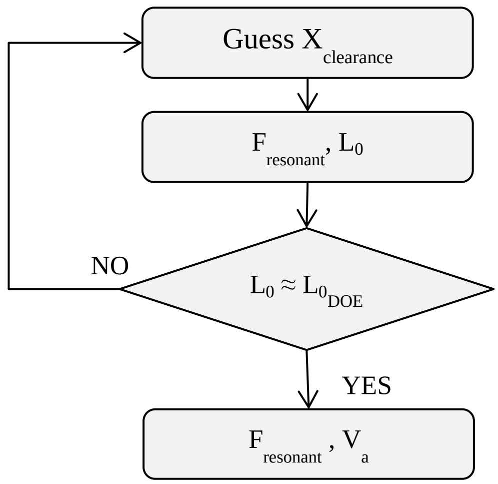


Figure 1: Resonance frequency seek algorithm for the analytical model

The above methodology was used to evaluate DoE sets quickly for different target cooling capacities and to look for main-effect parameters and subsequent optimization. The DoE used to highlight the efficacy of the analytical model consisted of 7 design parameters. Among these, 4 of them (mechanical spring stiffness, moving mass, geometric parameter and L_0) were set at 3 levels and the remaining 3 (valve port diameter, discharge and suction valve stiffnes) were set to 2 levels. These are shown in Figure 2, where the y-axis values correspond to the levels for each parameter when compared to the baseline compressor design value (highlighted by the big circles). Thus, a DoE set of around 700 runs with different geometric parameters was evaluated by the baseline model, and also with the IDF (integrated design framework/ integrated model) which is a system level transient simulation of the compressor operation [Mantri *et. al* (2014)].

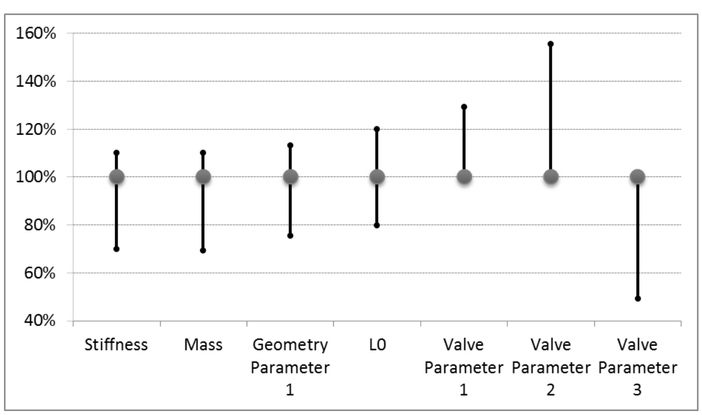
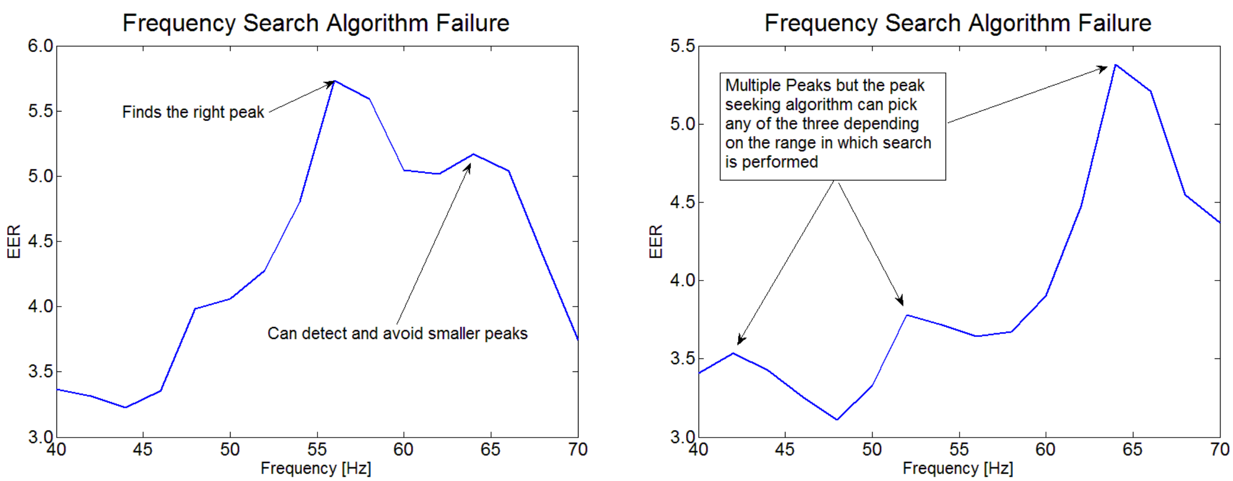


Figure 2: Parameter and their levels used in the design of experiments

The frequency search algorithm, as mentioned earlier, sweeps through a pre-set frequency band to search for the best EER. Running the DoE set shows that near the resonant frequency of the system, the performance parameter (here EER) is highly sensitive. Specifically, within an interval of about ±2 Hz, the EER could change by 0.6 points. Hence the stepping of the frequency during the search is important so as not to by-pass the resonant frequency. A starting point for the algorithm can be the resonant frequency of the mass suspended on the mechanical springs. The gas spring stiffness depends on the operating conditions and can cause the frequency shift by around 20Hz from the resonant frequency of the mechanical springs alone. Considering these factors, the algorithm used 3 stages of frequency search band windows with each window being obtained by narrowing the previous one. At the start, the band in which the resonance frequency was expected to lie was estimated and the algorithm was stepped with a fixed stepping to locate the resonant frequency. The search was then refined around this point. For example if the initial window was 40-70 Hz, the next window 2 was set at 50 to 65 Hz, and the last stage window 3 at 53 to 54 Hz. The system resonance frequency thus obtained after window 3 was 53.5 Hz. This helped avoiding a very fine stepping of 0.5 Hz for the very first band (40 – 70 Hz) to locate the resonant frequency. This standard algorithm works for most of the cases but is based on the assumptions that –

- The resonance frequency lies between the initial guess window
- There is only one maxima for EER in the initial guess window
- EER is initially increasing (at least for first two points) and then decreases

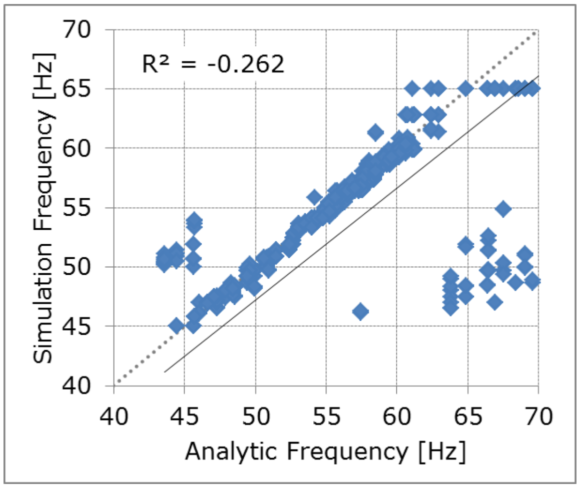


a. Converged correctly to 1st peak b. Converged incorrectly to 1st peak

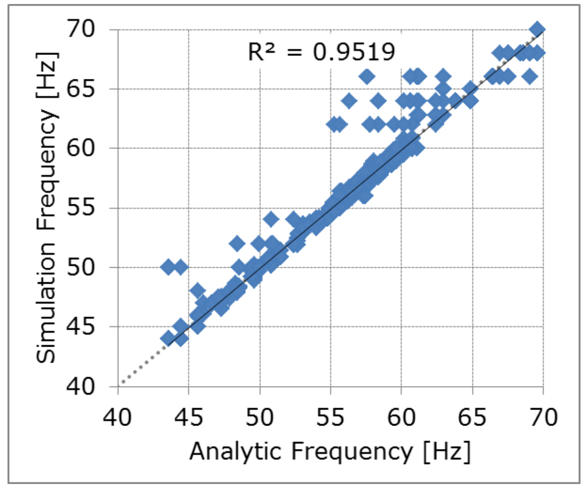
Figure 3: Few failure cases from the DoE with the presence of local maxima in the frequency sweep band when using the basic algorithm

Hence when running the DoE a further check on the estimated resonant frequency was ensured. For example a few cases from frequency search algorithm predicted a frequency with high phase angle between force and displacement. This is contrary to expectations because at resonance the expectation for this phase angle is to be close to zero. Further investigation of these cases by forcing a total sweep of frequency without using peak seeking algorithm was carried out. These were cases where the algorithm does not work because of presence of multiple peaks as shown in Figure 3 . Hence, in addition to the maximum EER point, the updated algorithm also monitors the phase angle and in the event of a high phase, a full-sweep setting for the frequency search is enforced.

The efficacy of the updated algorithm can be seen in the correlation between the frequency estimate from analytic model and the full transient simulation. Figure 4a shows, for another representative DoE set, that since the frequency search algorithm incorrectly estimated the resonant frequency by converging to the first local minima within the search band, the correlation between the analytic model and the full simulation is bad. Using the updated algorithm to also search for other maxima, when the phase angle criteria is unmet, the correlation was improved.

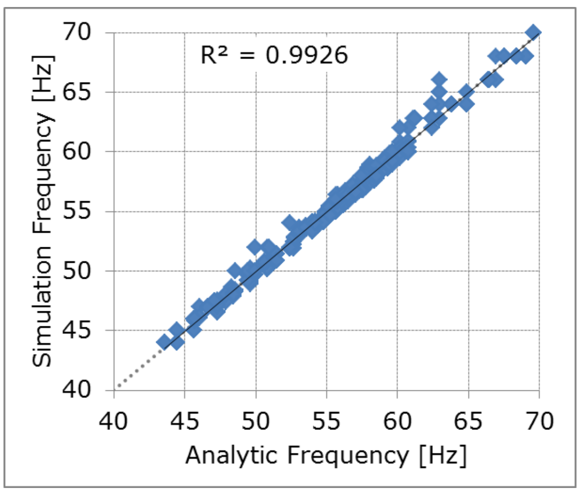


a. Using basic frequency search algorithm

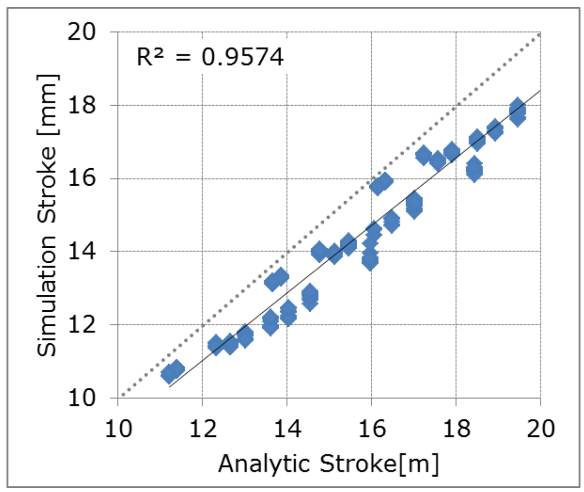


b. Using enhanced frequency search algorithm

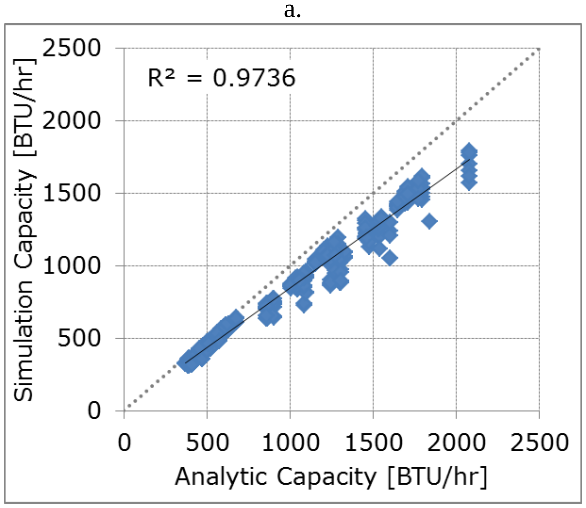
Figure 4: Resonant frequency estimation capability



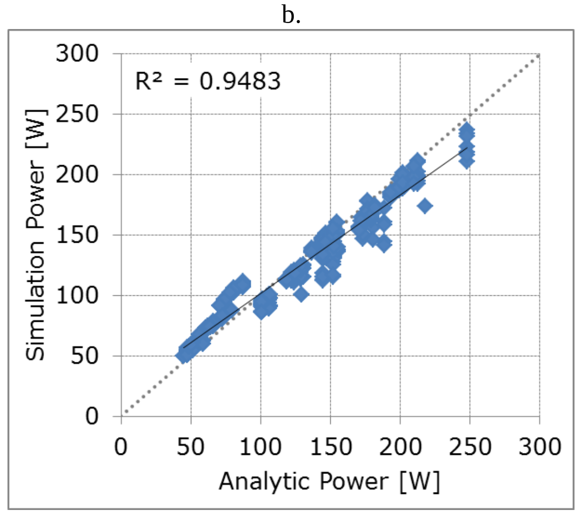
a.



b.



c.



d.

Figure 5: Comparing some performance parameters between the analytical (baseline) model and the multi-physics simulation (IDF) model.

3. RESULTS

The baseline model does not account for the valve losses and the effect of valve timing on the performance parameters. Most likely, the valve dynamics (timing) will affect the mass flow rate delivered and the input power but not the resonant frequency. As seen in Figure 5a, the resonant frequency prediction by the baseline model (also referred as the analytic prediction) matches well with the resonant frequency search algorithm within the IDF (also referred as the simulation prediction). This has a powerful implication of using the resonant frequency from the analytic closed form solution as the operational frequency in the full transient simulation, thus saving on 20X time. Since the analytical model does not consider the effect of valves, the cooling capacity is over predicted when compared to the simulation model, as is seen in Figure 5c. Similarly from Figure 5d, it is seen that the input power required to run the compressor at the operating conditions is not accurately predicted by the analytic model as the valve losses are not considered. But there is a correlation between the two capacity predictions and this is indicated by the R^2 value of the fit. This means, there is a possibility of incorporating the effect of valves on the cooling capacity by a valve efficiency factor. Additionally, the spread in the fit can be reduced by considering an analytic valve model for each design point.

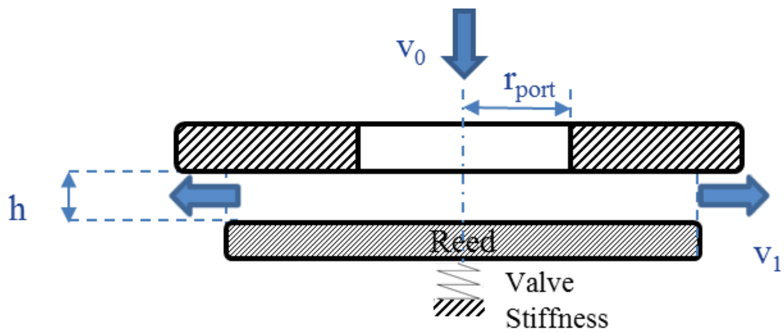


Figure 6: Flow through a valve

The valve losses manifest as a pressure drop across the valve during the suction or discharge stroke. Since, most of the contemporary refrigerator compressors use reed valves which are pressure actuated, the operation of valves require a pressure differential to open the valves to their lift positions and provide a required mass flow rate – *Q*. Consider a simple schematic of the flow through a valve port, shown in Figure 6, where the valve is open to a height *h*. The fluid velocity as it enters the valve port is v_0 and as it exits the valve is v_1. If the discharge coefficient of the valve is C_d, the flow rate through the valve can be represented by:

$$\mathrm{Q} = C_d A_{port} v_0 \tag{3}$$

where A_{port} is the area of the valve port. When the valve reed is close to the port opening, the resistance it offers to the flow is more, and this resistance decreases as the valve reed moves away (valve opens). It can be assumed that the discharge coefficient of the valve is linearly dependent on the reed displacement by the relation:

$$C_d = C_{d,0} \frac{h}{2r_{port}} \tag{4}$$

where $C_{d,0}$ is an empirical constant. Assuming that the valve reed is held in a static position at a displacement of *h*, the spring force then is exactly equal to the force on reed due to the fluid impinging on it. The valve displacement that is required to offer a flow rate of Q can then be given by:

$$\mathrm{h} = \left(\frac{4\rho Q^2}{\pi k C_{d,0}^2}\right)^{1/3} \tag{5}$$

where ρ is the density of the fluid when it enters the port and k is the valve stiffness. The pressure drop across the valve is related to the valve discharge coefficient by:

$$\Delta P = \frac{\rho}{2}\left(\frac{Q}{C_d A_{port}}\right)^2 \qquad 6$$

The increased work that needs to be done per cycle is the increased P-V work due to the pressure differential required to keep the reed valve open for the suction stroke duration. If the volume change of the cylinder chamber during the time that the valve is open is V_s, the valve loss is given by:

$$\text{Valve Loss} = \Delta P V_s F_{resonant} \qquad 7$$

Some reed valves within a reciprocating compressor (usually the discharge valves) have a limiting valve stop that controls the extent of the valve opening due to the high opening pressures during discharge. These might also be pre-stressed for optimized timing and efficient sealing. This then requires an additional loss component due to the pre-stress to be added to Equation 7.

Including these valve losses and pressure losses due to them as a part of the analytical model, the cooling capacity and power estimates were revised for the DoE. A comparison for these 2 performance parameters for the analytical model and IDF simulation is shown in Figure 7. When compared with the baseline model predictions in Figure 5, the updated analytical model which incorporates valve losses was able to predict the cooling capacity and input power closer to the IDF simulation estimates. The high R^2 values suggest that a correlation can be used in addition to the analytical model estimates to predict the cooling capacity and input power without running the time intensive multi-physics simulation.

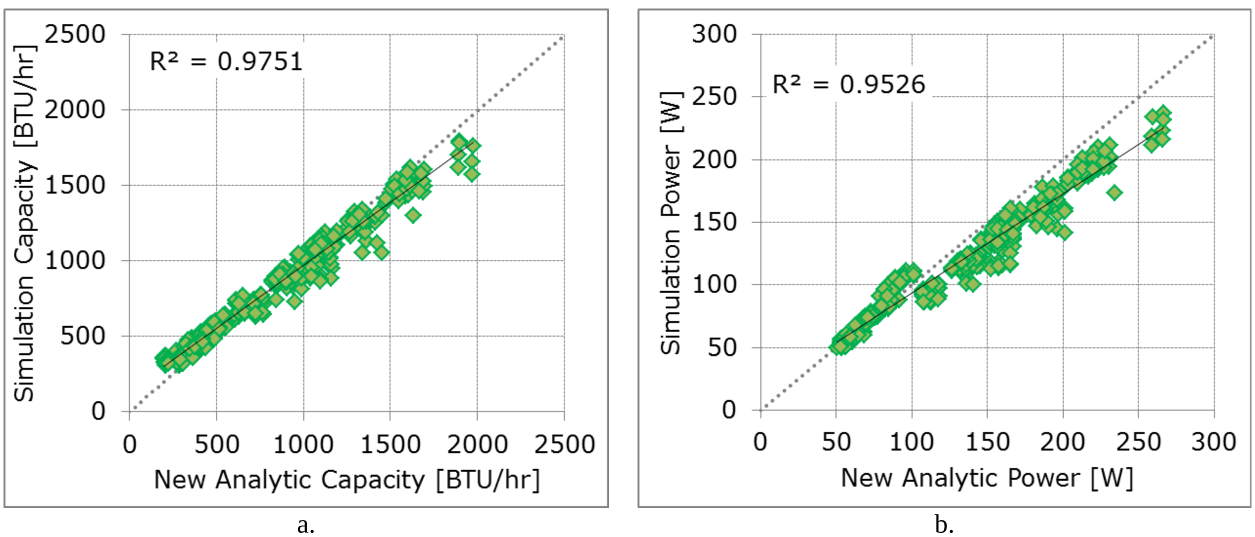


Figure 7: Comparison of compressor performance parameters between the updated analytical model and IDF

4. CONCLUSIONS

This study provides an extension to an existing analytical baseline model to better predict the compressor performance parameters like cooling capacity, input power, EER etc. The full multi-physics simulations are time intensive and limit the number of design parameters to include while performing a Design of Experiment (DoE) study for optimization. In such a scenario, an effective closed-form analytical model (instead of a transient simulation) is desired. The accuracy of the baseline model to estimate the resonant frequency of the system was compared for a sample DoE set with the full transient simulation. The resonant frequency of the system is accurately predicted and the iterative resonant frequency search of the system can be avoided in the transient simulation thus speeding up simulation 20X. Further improvements to the baseline model are suggested by incorporating loss terms due to compressor valves. An analytical formulation to estimate valve loss is highlighted, and using these loss terms the cooling capacity and power estimates were improved. A good correlation of analytical predictions with the full simulation estimates can also be used to estimate the EER.

REFERENCES

Choe, G., and Kim K., 2000, Analysis of nonlinear dynamics in a linear compressor, *JSME Int. Journal*, Series C, Vol. 43, No.3: 545 – 552.

Pollak, E., Soedel, W., Friedlaender, F. J. and Cohen, R., 1978, Mathematical model of an electro-dynamic oscillating refrigeration compressor, *International Compressor Engineering Conference at Purdue University*, West Lafayette, USA, 246 – 259.

Mantri, P., Bhakta, A., Mallampalli, S., Hahn, G., and Kusumba, S., 2014, Development and validation of integrated design framework for compressor system model, *Proc. of the 22nd International Compressor Engineering Conference at Purdue University*, West Lafayette, USA, 1153, 1-8.

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